Seminar Lecture

Introduction to Structural Estimation: A Stochastic Life Cycle Model

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Structural Estimation

- "Structural estimation is a methodological approach in empirical economics explicitly based on economic theory. A requirement of structural estimation is that economic modeling, estimation, and empirical analysis be internally consistent."
- "Structural estimation can be defined as theory-based estimation: the objective of the exercise is to estimate an explicitly specified economic model that is broadly consistent with observed data."

Why Structural Model?

- Theoretical model can not make quantitate predictions, even often can not make qualitive predictions.
- Numerical simulations rely on parameter assumptions. Its predictions are not reliable.
- Reduced form methods lack theoretical foundation, and can not make counterfactual simulation and welfare analysis.

Why Structural Estimation?

- With a structural model, we have two branches of method:
  - Calibration
  - Estimation: include MSM, SML, SMD, Indirect Inference, and Bayesian estimation.
- Estimation is more reliable than calibration.
- MSM (or SMM) is a general approach to estimate dynamic stochastic model.

Stochastic Life Cycle Model

- Stochastic life cycle model is a branch of general models that research agents life cycle behavioral, include consumption, wealth accumulation, health insurance, portfolio choice, retirement, tax policy etc.
- It perfectly connects economic theory, econometrics, and micro data.

For this seminar

- We will introduce a basic stochastic life cycle model and use the method of simulated moment (MSM) to estimate the model.
- We want to use this method to research the consumption effect and labor supply effect of the China Public Institutions Reform, analyze its welfare effect and do counterfactual simulation.

Preferences

- We assume that preferences take the form

$$u(C_t) + E_t\{\sum_{j=t+1}^{T} \beta^{j-t} u(C_j) + \beta^{T+1-t} b(A_{T+1})\}$$

- The within-period utility function is of the form

$$u(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$$

- workers who die value bequests of assets, A_t , according to the function

$$b(A_t) = \theta \frac{(A_t + k \cdot P_t)^{1-\rho}}{1-\rho}$$

Constraints

- The asset accumulation equation is

$$A_{t+1} = R(A_t - C_t) + Y_{t+1}$$

- where $A_t \geqslant 0$

Income Dynamic

- The income consist of permanent component P_t, and transitory component U_t

$$Y_t = P_t U_t$$

$$P_t = G_t P_{t-1} N_t$$

- where U_t and N_t are both independently and identically log-normally distributed, $lnU_t \sim N(0, \sigma_u^2)$, $lnN_t \sim N(0, \sigma_n^2)$,

Recursive Formulation

- In recursive form, the individuals problem can be written as

$$V_t(X_t) = max\{u(C_t) + \beta E_t V_{t+1}(X_{t+1})\}$$

- or

$$V_t(A_t, P_t) = max\{u(C_t) + \beta E_t V_{t+1}(A_{t+1}, P_{t+1})\}$$

- normalize the necessary variables by dividing them by permanent income, we can get:

$$V_t(a_t) = max\{u(c_t) + \beta E_t (G_{t+1} N_{T+1})^{1-\rho} V_{t+1}(a_{t+1})\}$$

Optimal Consumption Behavior

- use the permanent component of income to normalize

$$a_{t+1} = \frac{R(a_t - c_t)}{G_{t+1} N_{t+1}} + U_{t+1}$$

- The following Euler equation holds

$$u'(c_t(a_t)) = \beta R E[u'(c_{t+1}(a_{t+1}) G_{t+1} N_{t+1})]$$

- c_{T+1}, is linear in a_{T+1}:

$$c_{T+1} = \gamma_0 + \gamma_1 a_{T+1}$$

- The solution to the consumer problem consists of a set of consumption rules $\{c_t(a_t)\}_{1 \leqslant t \leqslant T}$

Gauss-Hermite Quadrature

- Gauss-Hermite Quadrature

$$u'(c_t(a_t)) = \beta R E[u'(c_{t+1}((a_t - c_t)\frac{R}{G_{t+1}N} + U)G_{t+1}N)]$$
$$\approx \sum_{i,j} f_t(n_i, u_j) w_{ij}$$

- where

$$f_t(n,u) = \frac{1}{\pi} u'(c_{t+1}((a_t - c_t)\frac{R}{G_{t+1}} e^{-\sqrt{2}\sigma_n n} + e^{\sqrt{2}\sigma_u u})G_{t+1} e^{\sqrt{2}\sigma_n n})$$

- $w_{i,j}$ are the weights , and n_i, u_j are nodes

The Method of Endogenous Gridpoints

- $x_t = a_t - c_t$, discretize x_t into $\{x_k\}(k = 1, 2...K)$
- get c_k^*, and $a_k^* = c_k^* + x_k$
- use linear interpolation method to get $\{c_t(a_t)\}$

Moment Conditions

- we employ a two-stage estimation procedure, we use additional data and moments to estimate χ in a first stage.
- by making the simulated moments as close as possible to theoretical mements

$$g_t(\theta;\hat{\chi}) = \frac{1}{N_t}\sum_{i=1}^{N_t} ln\hat{C}_{i,t}(\theta;\hat{\chi}) - ln\bar{C}_t$$

- Our second-stage estimation procedure is then a method of simulated moments estimator (MSM) that minimizes over θ:

$$\hat{\theta} = argmin\ g(\theta;\hat{\chi})'Wg(\theta;\hat{\chi})$$

Asymptotic Variance Covariance Matrix

- θ is is distributed asymptotically as normal distribution:

$$var(\hat{\theta}) = \frac{1+\tau}{N}(G_\theta' W G_\theta)^{-1} G_\theta' W V W G_\theta (G_\theta' W G_\theta)^{-1}$$

- And the statistic is distributed asymptotically as Chi-squared with $T - \#\theta$ degrees of freedom:

$$\chi = \frac{N}{1+\tau} g(\hat{\theta};\hat{\chi})' W g(\hat{\theta};\hat{\chi})$$

Equally-Weighted Minimum Distance Estimator

- Run a regression $lnY_{it} = f(X_{it}) + y_{it}$
- Decompose residual y_{it}

$$y_{it} = p_{it} + u_{it}, u_{it} \sim N(0, \sigma_u^2)$$

$$p_{it} = g_{it} + p_{it-1} + n_{it}, n_t \sim N(0, \sigma_n^2)$$

- Take first order difference

$$\Delta y_{it} = g_{it} + n_{it} + u_{it} - u_{it-1}$$

- Use the variance and covariance of Δy_{it} to generate moments for estimation

$$var(\Delta y_{it}) = \sigma_n^2 + 2\sigma_u^2$$

$$cov(\Delta y_{it}, \Delta y_{it-1}) = -\sigma_u^2$$

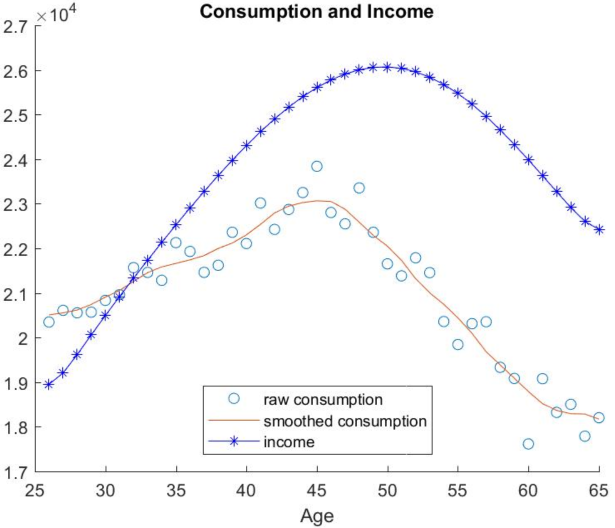
...

First-Step Parameter

Table 1

Parameter	Value
R	1.0344
σ_u^2	0.044
σ_n^2	0.0212
$\bar{w}$	-2.794
σ_w	1.784

Income and Cunsumption Profile

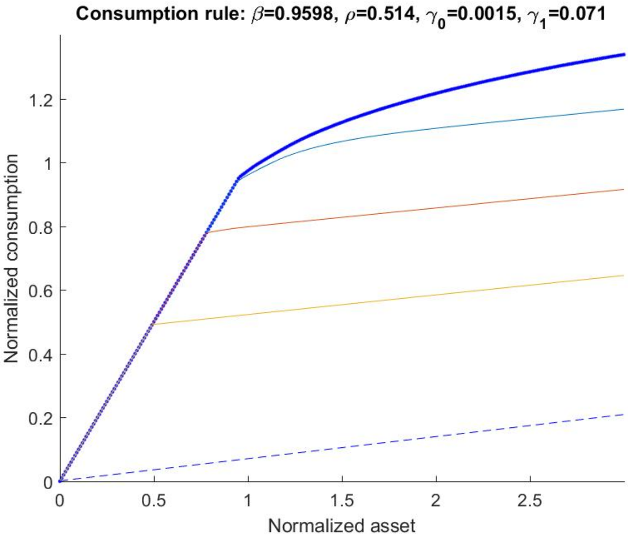


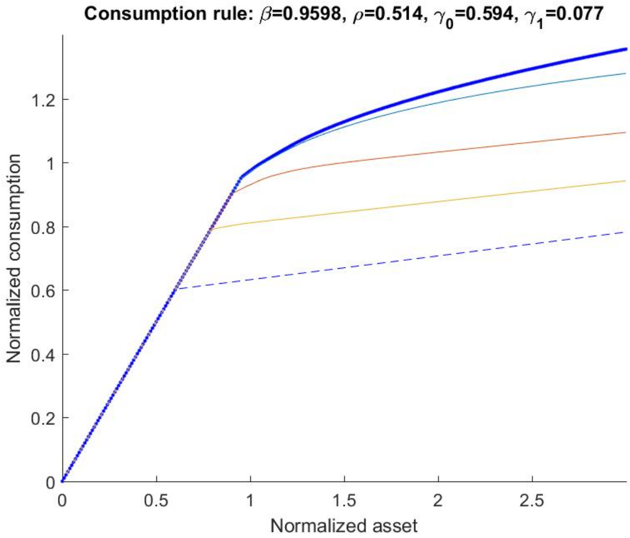
Estimation Result

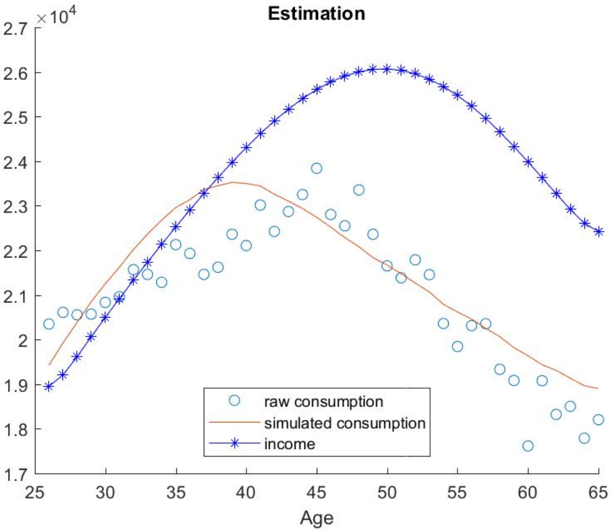
Table 2

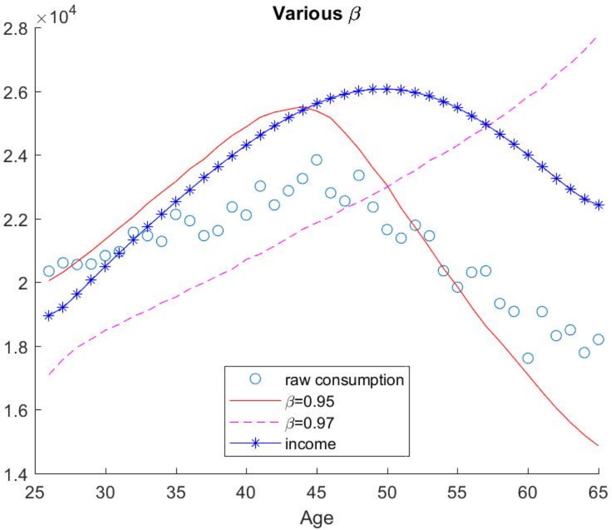
MSM Estimation	Equally Weighting
β	0.9596
	(0.0511)
ρ	0.5403
	(0.2140)
γ_0	0.0006
	(0.0002)
γ_1	0.0758
	(0.0273)
$\chi^2(36)$	128.8

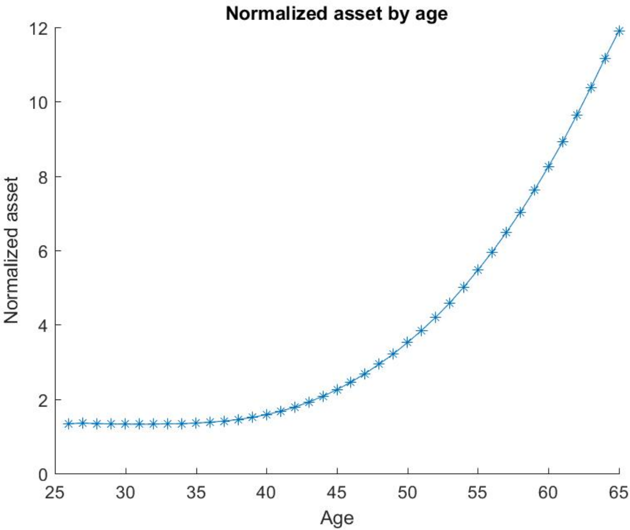
Standard errors in parentheses


Consumption rule: β=0.9598, ρ=0.514, γ0=0.0015, γ1=0.071
Normalized consumption
Normalized asset
0
0.2
0.4
0.6
0.8
1
1.2
0.5
1.5
2
2.5


Consumption rule: β=0.9598, ρ=0.514, γ0=0.594, γ1=0.077
Normalized consumption
Normalized asset
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5


Estimation
×10^4
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
1.7
25
30
35
40
45
50
55
60
65
Age
raw consumption
simulated consumption
income

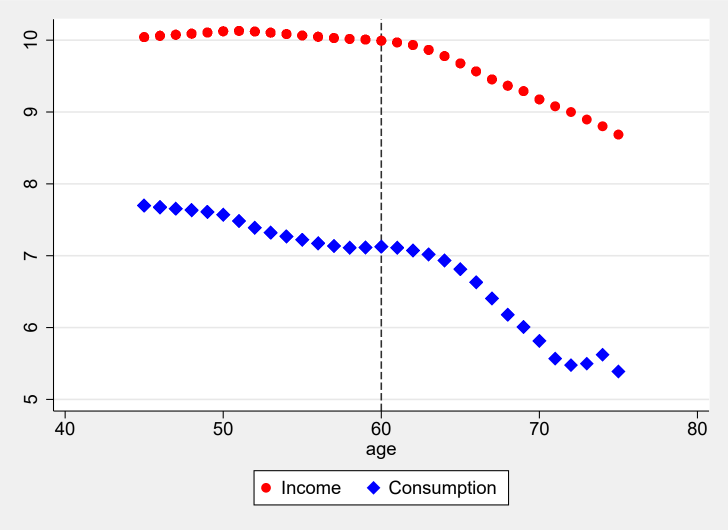

Various β
×10^4
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
25
30
35
40
45
50
55
60
65
Age
raw consumption
β=0.95
β=0.97
income


Normalized asset by age
Normalized asset
Age
0
2
4
6
8
10
12
25
30
35
40
45
50
55
60
65

Data

- CHARLS 2011-2013-2015 data
- hould comsunption C: exclude medical expenditure, education and training
- household income Y: household wage income
- keep $45 \leqslant age \leqslant 75$

Income and Consumption Profiles



Equally-Weighted Minimum Distance Estimator

- $\Delta y_{it} = (\Delta_2 y_{i,2013}, \Delta_2 y_{i,2015}, \Delta_4 y_{i,2015})'$
- Moment condition: $E(m_k m_j)$, $k, j \in \{1, 2, 3\}$
- Then we can get six moment condition

Missing Data

- $d_i = (d_{i,2011}, d_{i,2013}, d_{i,2015})$ and $d_{it} = 1\{\Delta y_{it}$ is not missing$\}$.
- we can drive

$$m = vech\{(\sum_{i=1}^{N} \Delta y_i \Delta y_i^{'}) \oslash (\sum_{i=1}^{N} d_i d_i^{'})\}$$

Health status and Medical expense

- The within-period utility function is of the form

$$u(C_t, H_t) = \delta(H_t)\frac{C_t^{1-\rho}}{1-\rho}$$

- Health status

$$\pi_{j,k} = P(H_{t+1} = k | H_t = j), j, k \in \{1, 0\}$$

- Survival rate s_t
- Medical expense M_t is

$$lnM_t = m(H_t, t) + \sigma(H_t, t)\psi_t$$

- government transfers

$$Tr_t = max\{0, \underline{C} + M_t - (A_t + Y_t)\}$$

Labor Supply

- The within-period utility function is of the form

$$u(C_t, L_t) = \frac{(C_t^{\gamma} L_t^{\gamma})^{1-\rho}}{1-\rho}$$

- The asset accumulation equation is

$$A_{t+1} = RA_t + W_t(L - L_t) - C_t$$

- wage dynamic

$$lnW_t = \alpha lnN_t + \omega_t$$

Portfolios Choice

- cash on hand evolves as

$$X_{t+1} = (1 + r^e_{t+1})S_t + (1 + r)B_t + Y_{t+1}$$

- the excess return of the risky asset is assumed to be i.i.d.:

$$r^e_{t+1} - r = \mu + \varepsilon_{t+1}$$

Reference

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