Supplementary Notes on Chapter 5 of D. Romer's Advanced Macroeconomics Textbook (4th Edition)

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Interpreting ρ_A and ρ_G

$$(5.9) \qquad \tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_{A,t}, \ -1 < \rho_A < 1$$
$$(5.11) \qquad \tilde{G}_t = \rho_G \tilde{G}_{t-1} + \varepsilon_{G,t}, \ -1 < \rho_G < 1$$

- Merits of Stationary processes? Effects of a sudden shock keep diminishing. Extreme case: $\rho = 0$.
- $E(\tilde{A}_t) = E(\tilde{G}_t) = 0$.
- Q: Considering the Endogenous Growth ingredients, $\rho_A \lesseqqgtr 0$?
- Q: Considering the Government spending rules, $\rho_G \lesseqqgtr 0$?

Interpreting Equation (5.24)

$$(5.23) \qquad \frac{1}{c_t} = e^{-\rho} E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$(5.24) \qquad \frac{1}{c_t} = e^{-\rho} \left[E_t \left(\frac{1}{c_{t+1}} \right) E_t(1 + r_{t+1}) + Cov \left(\frac{1}{c_{t+1}}, 1 + r_{t+1} \right) \right]$$

- Euler Equation with uncertainty.
- $Cov(X, Y) = E[(X - E(X))(Y - E(Y))]$. Interpreted as how the patterns of the realization bundles of c_{t+1} and r_{t+1} (they are both random variables) would look like.
- If $Cov < 0$, households know that it is more likely (compared to the baseline case where $Cov = 0$) that if a higher next-period interest rate occurs, a lower next-period marginal utility (on consumption) also occurs. As a result, given other conditions fixed, i.e, given $E_t(\frac{1}{c_{t+1}})$ and $E_t(1 + r_{t+1})$, the households want to save less (higher c_t).
- What if $Cov > 0$? Economic intuitions.

Interpreting Equation (5.31)

$$(5.23) \qquad \frac{1}{c_t} = e^{-\rho} E_t \left[\frac{1}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$(5.31) \qquad \ln s_t - \ln(1 - s_t) = -\rho + n + \ln \alpha + \ln E_t \left(\frac{1}{1 - s_{t+1}} \right)$$

- If households choose $\hat{s} = \ln \alpha + n - \rho$ in each period, then there is no uncertainty in s_{t+1} and $\hat{s} = s_t = s_{t+1}$ solves (5.31).
- Economic intuition? In this specific and oversimplified model (with $\delta = 0$ and $G_t \equiv 0$), the technological shock, say a positive one, leads to both a decrease in $\frac{1}{c_{t+1}}$ (given other things fixed) and an increase in r_{t+1} (why?). Magically, in this model, the two effects offset each other:

Interpreting Equation (5.31) (Continued)

$$\frac{1+r_{t+1}}{c_{t+1}} = \frac{\alpha\left(\frac{A_{t+1}L_{t+1}}{K_{t+1}}\right)^{1-\alpha}}{(1-s_{t+1})Y_{t+1}\cdot\frac{1}{N_{t+1}}} = \frac{\alpha Y_{t+1}K_{t+1}^{-1}}{(1-s_{t+1})Y_{t+1}}.$$

- The two Y_{t+1}'s offset each other. According to (5.27), there is no uncertainty in K_{t+1} at time t. As a result, if s_{t+1} is fixed at $\hat{s}$, there is no need for the households to adjust c_t according to the contingent technological shock $\tilde{A}_{t+1}$.
- Any other equilibrium paths? There may be, but isn't an equilibrium path with $s_t \equiv \hat{s}$ beautiful and intuitive?

Interpreting Equation (5.37)

- it is thus straightforward to understand equation (5.37), given $\frac{c_t}{1-\ell_t} = \frac{w_t}{b}$ (5.26), $c_t = Y_t \frac{1-\hat{s}}{N_t}$, $w_t = Y_t \frac{1-\alpha}{\ell_t N_t}$: First, ℓ_t does not need to adjust according to $\tilde{A}_{t+1}$. Second, shock $\tilde{A}_t$ does not change ℓ_t, a current-period positive technological shock tends to increase income and to decrease marginal utility on consumption, these two effects offset each other. Consequently, we have $\ell_t \equiv \hat{\ell} = \frac{1-\alpha}{1-\alpha+b(1-\hat{s})}$.
- Again, we get fixed $\hat{\ell}$ and $\hat{s}$ only in this oversimplified model!

Interpreting Fluctuations in Y_t

$$
\begin{aligned}
(5.39)\ \ln Y_t = \quad & \alpha \ln \hat{s} + \alpha \ln Y_{t-1}^{trend} + (1-\alpha)(\bar{A} + gt + \ln \hat{\ell} + \bar{N} + nt) \\
& + \alpha \tilde{Y}_{t-1} + (1-\alpha)\tilde{A}_t \\
= \quad & \ln Y_t^{trend} + \tilde{Y}_t \ ,
\end{aligned}
$$

where

$$
\ln Y_0^{trend} = \alpha \ln K_0 + (1-\alpha)(\bar{A} + \ln \hat{\ell} + \bar{N}) \quad (1)
$$

$$
\tilde{Y}_0 = (1-\alpha)\tilde{A}_0 = (1-\alpha)\varepsilon_{A,0} \quad (2)
$$

$$
(5.42) \quad \tilde{Y}_t = (\alpha + \rho_A)\tilde{Y}_{t-1} - \alpha\rho_A \tilde{Y}_{t-2} + (1-\alpha)\varepsilon_{A,t}
$$

- Numerical examples: $\alpha = \frac{1}{3}, \rho_A = \frac{1}{3}, \bar{A} = 10, g = 5, b = \frac{1}{2}, n = 0.2, \rho = 0.1, \bar{N} = 10, K_0 = 10,\ Var(\varepsilon_{A,t}) = 10.$

Interpreting Fluctuations in Y_t (Continued)

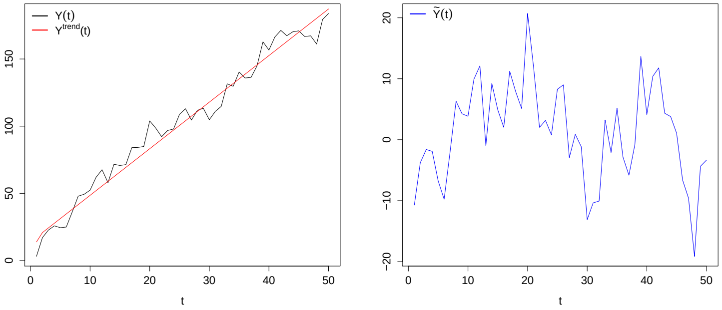


Figure 1: Numerical example with $\alpha = \frac{1}{3}$, $\rho_A = \frac{1}{3}$, $\bar{A} = 10$, $g = 5$, $b = \frac{1}{2}$, $n = 0.2$, $\rho = 0.1$, $\bar{N} = 10$, $K_0 = 10$, and $Var(\varepsilon_{A,t}) = 10$.

The oversimplified model, Pros and Cons

Pros

- Analytically solvable.
- Delivers the idea of **Real Fluctuations** under parsimonious settings.

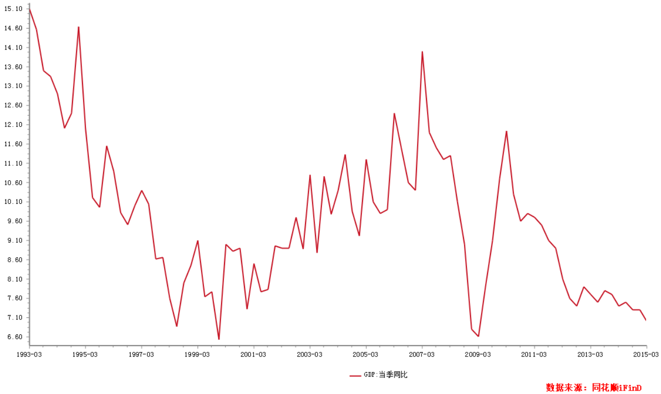
Cons

- Unrealistic predictions. Recall that $s(t) \equiv \hat{s}$, so consumptions and investments fluctuate at the same rate as the output does. Table 5.2...

An Example

$\alpha = \frac{1}{3}, g = 0.005, n = 0.0025, \delta = 0.025, \rho_A = 0.025, \rho_G = 0.95, r^* = 0.015, \ell^* = \frac{1}{3}$.
$a_{LA} = 0.35, a_{LK} = -0.31, a_{CA} = 0.38, a_{CK} = 0.59, b_{KA} = 0.08, b_{KK} = 0.95, a_{CG} = -0.13, a_{LG} = 0.15, b_{KG} = -0.004$.
For China, reset $b_{KG} = 0.4 * 0.95 + 0.6 * (-0.004)$. (Why?)

$$
\begin{aligned}
\tilde{Y}_t &= \ln Y_t - \ln Y_t^{trend} \\
&= \ln \frac{Y_t}{Y_{t-1}} - \ln \frac{Y_t^{trend}}{Y_{t-1}} \\
&\simeq g_Y(t) - g_Y^{fundamental}(t)
\end{aligned}
$$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1999-03
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2003-03
2005-03
2007-03
2009-03
2011-03
2013-03
2015-03
GDP:当季同比
数据来源：同花顺iFinD

Figure 2

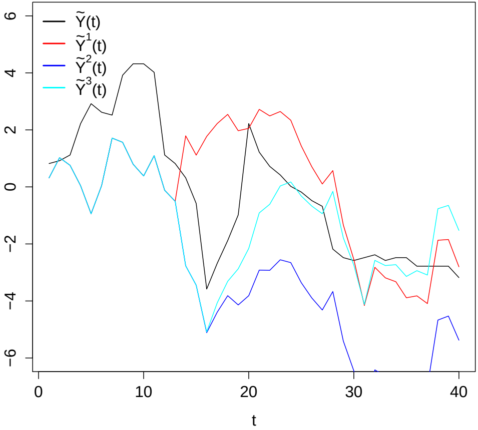

Figure 3: Period: 2nd quarter of 2005 - 1st quarter of 2015. $\tilde{Y}$ is the real outcome, $\tilde{Y}^1$ neglects the 2008 financial crisis and 2009 bailout plans, while $\tilde{Y}^2$ considers the former and $\tilde{Y}^3$ further considers the latter.