

Applied Game Theory

Graduate Program in Economics, HUST

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HOMEWORK #3

(Should you find any mistakes or typos in this file, please do not hesitate to contact me, at yiming@hust.edu.cn)

1. Consider a *symmetric* Cournot game with $n \geq 2$ firms. Specifically, let $a > 0$ and let the inverse demand function be given by

$$P(Q) = \begin{cases} a - Q, & \text{for } 0 \leq Q \leq a; \\ 0, & \text{for } Q > a. \end{cases}$$

The marginal cost for each firm is $c_i \equiv c \ll a$. It chooses output $q_i \geq 0$ and incurs costs $c_i \cdot q_i$. Total or industry output is $Q = \sum_{j=1}^n q_j$. Firm i 's payoff (profit) is $\pi_i(q_1, \dots, q_n) = P(Q) \cdot q_i - c_i \cdot q_i$.

- (a) Find the unique Nash Equilibrium for the game above. What is the total profit of the n firms?
- (b) Suppose the n firms are merged into a single monopoly, what is the total profit of the newly established firm? Is it greater than that in (a)?
- (c) Suppose the firms are not interested in being merged into a single big firm. However, they all join a organization, i.e., a cartel, to coordinate their output levels. Prove the cartel is *unstable* in the sense that the output plan maximizing the total profit designed by the cartel cannot constitute a Nash Equilibrium.
- (d) All the questions above are based on a *static* setting of the game above. Now suppose the Cournot game is played **infinitely many times**. Is it possible for all the firms to achieve an SPNE in which the total profit is the same as that of the monopoly in (b), in each *stage game*? If yes, find one such SPNE. If no, prove it. (Hint: Recall the second Folk Theorem.)

2. Consider the dynamic game with imperfect information illustrated in Figure 1.

- (a) Show that it is possible to have (L, r) as part of a Perfect Bayesian Nash Equilibrium (PBE) of the game. Give an example of it and describe the specific belief system prescribed by the PBE.
- (b) Show that your answer in (a) is also an SPNE.
- (c) Show that your answer in (a) is not a Sequential Equilibrium.
- (d) Find a Sequential Equilibrium of the Game.

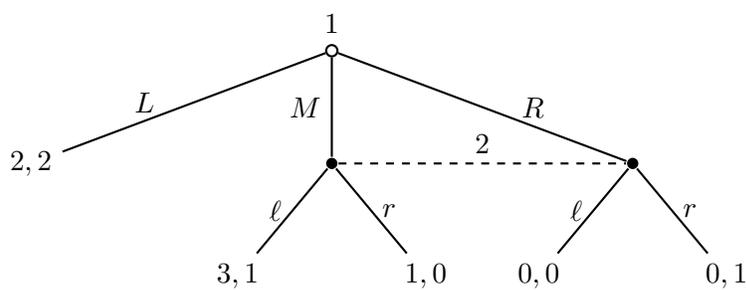


Figure 1: A dynamic game with imperfect information.