Applied Game Theory
Graduate Program in Economics, HUST
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Fall 2018
HOMEWORK #2
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1. Consider the following normal form game with incomplete information.

   - Player set: $I = \{1, 2\}$.
   - Strategy sets: $S_1 = \{T, B\}$, $S_2 = \{L, R\}$.
   - Type sets: $T_1 = \{t_{11}, t_{12}\}$, $T_2 = \{t_{21}, t_{22}, t_{23}\}$.
   - Probability distributions (or beliefs):

$$
\begin{aligned}
&P(t_{21}|t_{11}) = 1,\ P(t_{22}|t_{11}) = 0,\ P(t_{23}|t_{11}) = 0;\\
&P(t_{21}|t_{12}) = 0,\ P(t_{22}|t_{12}) = 1,\ P(t_{23}|t_{12}) = 0;\\
&P(t_{11}|t_{21}) = \frac{2}{5},\ P(t_{12}|t_{21}) = \frac{3}{5};\\
&P(t_{11}|t_{22}) = \frac{1}{2},\ P(t_{12}|t_{22}) = \frac{1}{2};\\
&P(t_{11}|t_{23}) = \frac{3}{4},\ P(t_{12}|t_{23}) = \frac{1}{4}.
\end{aligned}
$$

   - Payoff matrices without incomplete information are given as in Table 1:

Table 1: Payoff matrices.

	L	R
T	$0, -1$	$2, 1$
B	$2, 1$	$0, -1$

(i) Type $t = (t_{11}, t_{21})$.

	L	R
T	$0, -2$	$2, 0$
B	$2, 4$	$0, 2$

(ii) Type $t = (t_{11}, t_{22})$.

	L	R
T	$0, -1$	$2, 1$
B	$2, 0$	$0, -1$

(iii) Type $t = (t_{11}, t_{23})$.

	L	R
T	$2, -1$	$2, 1$
B	$0, 1$	$0, -1$

(iv) Type $t = (t_{12}, t_{21})$.

	L	R
T	$2, -2$	$2, 0$
B	$0, 4$	$0, 2$

(v) Type $t = (t_{12}, t_{22})$.

	L	R
T	$2, -1$	$2, 1$
B	$0, 0$	$0, -1$

(vi) Type $t = (t_{12}, t_{21})$.

Determine all Bayesian Equilibria (BNE in pure strategies) of the game.

2. Consider the Cournot duopoly with firms $j = 1, 2$ and

   - constant marginal costs $c_1 = c_2 = c$;
   - linear inverse demand function $P(y) = a - by$ for $y \leq \frac{a}{b}$;
   - $a > c$.

   In this case, firm 2's best response against y_1 is given by

$$y_2(y_1) = \frac{1}{2b}\left(a - c - by_1\right), \qquad (1)$$

   if $a - c - by_1 \geq 0$.

   Suppose the two firms play a **two-stage** game where firm 1 moves first and firm 2 moves second. Let $\bar{y}_1 \in (0, \frac{a-c}{b})$ and the firms' strategies in the two-stage game be given by

$$y_1 = \bar{y}_1, \qquad (2)$$

$$y_2(y_1) = \begin{cases} \frac{1}{2b}(a - c - b\bar{y}_1), & \text{if } \; y_1 = \bar{y}_1; \\ \frac{a-c}{b}, & \text{if } \; y_1 \neq \bar{y}_1. \end{cases} \qquad (3)$$

   Show that strtegies depicted in equations (2) and (3) constitute a Nash equilibrium of the two-stage game, which is not subgame perfect.